Name Jamile Ali Kaddoura, ID G00097411

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MTH 111, Exam I

Ayman Badawi

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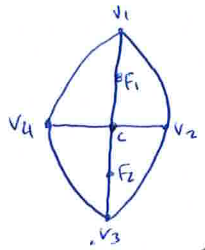
QUESTION 1. (8 points) Consider the ellipse $\frac{(x-3)^2}{7} + \frac{(y+2)^2}{16} = 1$

(i) Sketch roughly.



$\left(\frac{k}{2}\right)^2 = 16$ $\quad b^2 = 7$

$\frac{k}{2} = 4$ $\quad c = (3,-2)$

$k = 8$

(ii) Find the ellipse constant.

$k = 8$

(iii) Find all 4 vertices.

$V_1 = (3,2)$ / $V_2 = (5.6, -2)$ or $V_2 = (3+\sqrt{7}, -2)$ / $V_3 = (3,-6)$ $V_4 = (3-\sqrt{7}, -2)$ or $V_4 = (0.35, -2)$

$V_1 = (3,2)$

$V_2 = (3+\sqrt{7}, -2)$

$V_3 = (3,-6)$

$V_4 = (3-\sqrt{7}, -2)$

$|CF_1| = \sqrt{\left(\frac{k}{2}\right)^2 - b^2}$

$\sqrt{16-7}$

$|CF_1| = \sqrt{9} = 3$

This is foci → (iv) Find the focus.

$F_1 = (3,1)$ $\quad F_2 = (3,-5)$

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QUESTION 2. (8 points) Consider the parabola: $y = 5x^2 + 40x + 82$

(i) Write it in the standard form.

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$y = 5(x^2 + \cancel{40}\,8) + 82$

$y = 5(x+4)^2 - 16 + 16 + 82$

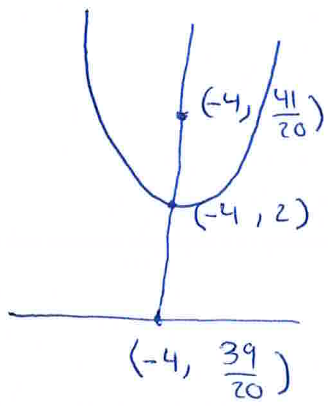
$y = 5(x+4)^2 - 80 + 82$

$y = 5(x+4)^2 + 2$

$(y-2) = 5(x+4)^2$

$\frac{1}{5}(y-2) = (x+4)^2$

(ii) Sketch roughly.



$4d = \frac{1}{5}$

$d = \frac{1}{20}$

$y = \frac{39}{20}$

(iii) Find the focus

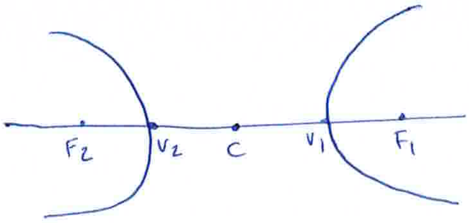
$\left(-4, \frac{41}{20}\right)$

(iv) Find the equation of the directrix.

$y = \frac{39}{20}$

QUESTION 3. (8 points) Consider the hyperbola $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{12} = 1$

(i) Sketch roughly.



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$(\frac{k}{2})^2 = 4 \quad \frac{k}{2} = 2 \quad k = 4$

$b^2 = 12 \quad b = \sqrt{12}$

$c = (2, -3)$

$|CF_1| = \sqrt{(\frac{k}{2})^2 + b^2}$

$|CF_1| = \sqrt{4 + 12}$

$|CF_1| = \sqrt{16} = 4$

(ii) Find the hyperbola-constant

$k = 4$

(iii) Find the foci.

$F_2 = (-2, -3) \quad F_1 = (6, -3)$

(iv) Find the vertices (two of them).

$V_2 = \cancel{(2, -5)} \quad V_1 = \cancel{(2, -1)}$

$V_2 = (0, -3) \quad V_1 = (4, -3)$

QUESTION 4. (6 points) Find the parametric equations of the line that passes through $(1, 4, 6)$ and $(6, 8, 7)$. Then find the symmetric equation.

$\langle 5, 4, 1 \rangle \qquad (1, 4, 6) + \langle 5t, 4t, 1t \rangle$

Parametric equation =

$x = 5t + 1$

$y = 4t + 4$

$z = 1t + 6$

Symmetric equation =

$\frac{x-1}{5} = \frac{y-4}{4} = \frac{z-6}{1}$

or

$\frac{x-1}{5} = \frac{y-4}{4} = z - 6$

QUESTION 5. Let $V = \langle 1, 5, 2 \rangle$ and $W = \langle 2, 6, 5 \rangle$. Find the projection of W over V

$\mathrm{Proj}_V^W = \frac{V \cdot W}{|V|^2} \cdot V$

$\langle 1, 5, 2 \rangle \cdot \langle 2, 6, 5 \rangle \rightarrow 2 + 30 + 10 = 42$

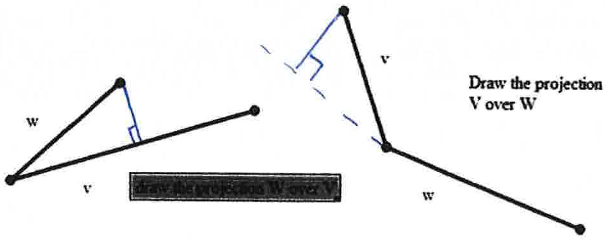
$|V|^2 = \sqrt{(1)^2 + (5)^2 + (2)^2}^2$

$1 + 25 + 4 = 30$

$\frac{42}{30} \cdot \langle 1, 5, 2 \rangle$

$\mathrm{Proj}_V^W \rightarrow \langle \frac{42}{30}, 7, \frac{14}{5} \rangle$

QUESTION 6. (4 points)



QUESTION 7. (8 points) Let $L_1 : x = 2t + 1, y = -3t + 2, z = 2t$ and $L_2 : x = w - 1, y = 2w - 9, z = 2w - 6$

If L_1 is perpendicular to L_2, find the intersection point.

$D_1 = \langle 2, -3, 2 \rangle \quad D_2 = \langle 1, 2, 2 \rangle$

$2 - 6 + 4 = 0 \rightarrow$ they are perpendicular
dot product $= 0$

L_1

$x = 2t + 1$
$y = -3t + 2$
$z = 2t$

L_2

$x = w - 1$
$y = 2w - 9$
$z = 2w - 6$

$2t + 1 = w - 1 \rightarrow 2t - w = -1 - 1 \rightarrow 2t - w = -2$
$-3t + 2 = 2w - 9 \rightarrow -3t - 2w = -2 - 9 \rightarrow -3t - 2w = -11$
$2t = 2w - 6 \rightarrow 2t - 2w = -6$

t	w
-2	-1
-11	-2
t	w
+2	-1
-3	-2

$$\frac{(-2)(-2) - (-1)(-11)}{(-2)(+2) - (-1)(-3)} = \frac{4 - 11}{-4 - 3} = \frac{-7}{-7} = \cancel{7}\,1$$

$t = \cancel{7}\,1$

$\cancel{-2(7) - w = -2}$
$\cancel{14 - w = -2}$
$\cancel{14 + 2 = w = 16}$

$\cancel{2(7) - 2(16) = -6}$
$\cancel{-14 - 32}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

$2(1) - w = -2$
$2 - w = -2$
$2 + 2 = w$
$4 = w$

$2 - 2(4) = -6$
$2 - 8 = -6$
$-6 = -6$

intersection =

$x = 2(1) + 1 = 3$
$y = -3(1) + 2 = -1$
$z = 2$

intersection point

$(3, -1, 2)$